

Space anisotropy search at colliders

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Abstract

In the framework of model with Lorentz violation (LV) we discuss a physical observables for $q\bar{q}$ pair production at lepton-lepton colliders and describe the experimental signal to be detected. We obtain a conservative limits on Lorentz-violating dimensionless coupling for quark sector from LEP data. We also make a phenomenological prediction for LV model at the future lepton collider.

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1 Introduction

The problem of space-time anisotropy is a great challenge of high energy physics. The attempts to measure the space anisotropy for a relatively low energy scale are widely performed by an astrophysical experimental searches. Nevertheless, the ability to search fundamental properties of space-time on a high energy scales appears with LHC launching. A violation of Lorentz invariance is one of the possible reason of space anisotropy. There are various self-consistent setups of quantum gravity which admit the violation of Lorentz invariance: a models of quantum loop gravity [1, 2], string model setups [3, 4], a models of Horava-Lifshitz with extra spatial derivatives [5, 6, 7], the models of the analogue gravity [8]. The most general Lorentz-violating Lagrangian with gauge invariant renormalizable terms was performed by [9, 10] for the particles of standard model (SM). The former framework is known as Standard-Model Extension (SME) of Alan Kostelecky. The current constraints on SME parameters are presented in [11]. In particular, the limits for leptons have been set at the level of $10^{-6} - 10^{-20}$.

A very recent result [12] claims that CPT -even coefficients for LV in the quark sector (e.g. for u and d quarks) can be bounded at the level about $10^{-5} - 10^{-6}$ from HERA experimental data on deep inelastic scattering (DIS) of e^-p . A framework of SME has been explored carefully also in the context of Tevatron collider phenomenology. In particular Ref. [13] provides the bounds on CPT -even LV couplings of top quark from dependence of the $t\bar{t}$ production cross-section on sidereal time as the orientation of the D0 detector changes with the rotation of the Earth. Test of CPT -odd symmetry violation for B -mesons was performed in Refs. [14, 15].

However the limits on CPT -even LV coupling ($c_{Q(U,D)}\epsilon_{ZZAB}$) for quarks hasn't been obtained yet. In the present paper we discuss a possible implication of the SME phenomenology for quark

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sector at a lepton-lepton colliders. Namely, we calculate the production cross-section of $q\bar{q}$ pair via γ and Z^0 -boson for the SME couplings that affect the quark field.

The paper is organized as follows. In Sec. 2 we consider general Lagrangian of the SME and perform LV couplings of quarks to be constrained by collider experiment. In Sec. 3 we derive the matrix element squared for the process $e^+e^- \rightarrow q\bar{q}$ in SME. In Sec. 4 we consider the spatial transformations from a Sun-centered reference frame to the Earth-based laboratory frame. In Sec. 5 we obtain a very conservative limits for LV coupling of u, d, s, c and b quarks from ALEPH and OPAL data. In Sec. 6 we derive time-dependent cross-section for the process $e^+e^- \rightarrow q\bar{q}$ and make SME prediction for the future lepton-lepton collider.

2 SME Lagrangian

We begin with a general SME Lagrangian, which can be expressed in the following form

$$\mathcal{L}_{SME} = \mathcal{L}_{SM} + \mathcal{L}_{LV}, \quad (1)$$

where \mathcal{L}_{SM} is the standard model (SM) Lagrangian and \mathcal{L}_{LV} contains renormalizable Lorentz-violating terms for SM fields. Now let us consider CPT even Lagrangian for the quark sector

$$\begin{aligned} \mathcal{L}_{LV} \supset \mathcal{L}_{LV}^{quarks} = & i(c_Q)_{\mu\nu AB} \bar{Q}_A \gamma^\mu D^\nu \bar{Q}_B + i(c_U)_{\mu\nu AB} \bar{U}_A \gamma^\mu D^\nu \bar{U}_B + \\ & + i(c_D)_{\mu\nu AB} \bar{D}_A \gamma^\mu D^\nu \bar{D}_B, \end{aligned} \quad (2)$$

where index A labels the quark flavor, $A = 1, 2, 3$, here $u_A = (u, c, t)$ and $d_A = (d, s, b)$. We denote left- and right-handed quarks in (2) by $Q_A = (u_A, d_A)_L$, $U_A = (u_A)_R$ and $D_A = (d_A)_R$. The dimensionless LV coefficients $(c_Q)_{\mu\nu AB}$, $(c_U)_{\mu\nu AB}$ and $(c_D)_{\mu\nu AB}$ can be assumed symmetric in flavor indices, A, B and traceless in space-time indices, μ, ν . For definiteness in the present paper we consider a very specific case of Lorentz violation instead of treating full SME Lagrangian (2) for quarks, when $A = B$. Namely, the subjects of our interest are the Lagrangians for quarks in the $SU(2) \times U(1)$ breaking sector, $\mathcal{L}_{LV} = \sum_q (\mathcal{L}_{LV}^{\gamma\bar{q}q} + \mathcal{L}_{LV}^{Z\bar{q}q})$. As an illustration we perform below the lagrangian for b quark

$$\mathcal{L}_{LV}^{\gamma\bar{b}b} = Q_b e \bar{b} \left(c_{Q\mu\nu} \frac{(1 - \gamma_5)}{2} + c_{D\mu\nu} \frac{(1 + \gamma_5)}{2} \right) \gamma^\mu b A^\nu \quad (3)$$

$$\mathcal{L}_{LV}^{Z\bar{b}b} = \frac{e}{\sin 2\theta_W} \bar{b} \left(c_{Q\mu\nu} C_L^f \frac{(1 - \gamma_5)}{2} + c_{D\mu\nu} C_R^f \frac{(1 + \gamma_5)}{2} \right) \gamma^\mu b Z^\nu \quad (4)$$

here we denote for simplicity $c_{Q\mu\nu} \equiv c_{Q\mu\nu 33}$ and $c_{D\mu\nu} \equiv c_{D\mu\nu 33}$. For other flavors only diagonal elements have been left, say, for c -quark we have $c_{Q\mu\nu} \equiv c_{Q\mu\nu 22}$ and $c_{U\mu\nu} \equiv c_{U\mu\nu 22}$. These coefficients to be constrained by the collider experiment. All remaining LV coefficients for quarks in (2) can be set to zero without loss of generality. We also use a convenient SM notations $C_L^f = 2T_3^f - 2Q_f \sin^2 \theta_W$ and $C_R^f = -2Q_f \sin^2 \theta_W$ in (3) and (4).

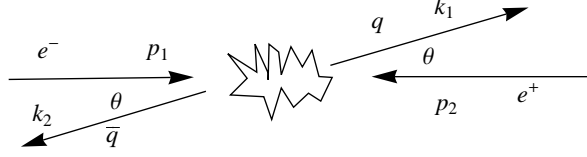


Figure 1: the diagram illustrates the kinematics of the process $e^+e^- \rightarrow q\bar{q}$.

3 The matrix element for SME setup

In this section we calculate the matrix element squared for the signal process $e^+e^- \rightarrow q\bar{q}$ at lepton-lepton collider for the case of Lorentz violation (3) and (4). The amplitude squared, which corresponds to $q\bar{q}$ pair production via γ and Z^0 boson can be written as sum of SM term and SM-SME interference terms in the leading order of LV couplings $c_{\mu\nu}^Q$, $c_{\mu\nu}^U$ and $c_{\mu\nu}^D$

$$\sum_{s.c.} |\mathcal{M}(e^+e^- \rightarrow q\bar{q})|^2 \simeq \underbrace{\sum_{s.c.} |\mathcal{M}_\gamma + \mathcal{M}_Z|^2}_{|M|_{SM}^2} + \underbrace{\sum_{s.c.} \left(2\mathcal{M}_\gamma^\dagger \delta\mathcal{M}_\gamma + 4\mathcal{M}_\gamma^\dagger \delta\mathcal{M}_Z + 2\mathcal{M}_Z^\dagger \delta\mathcal{M}_Z \right)}_{\delta|M|_{SME}^2}, \quad (5)$$

in the expression above we average the amplitude squared over the initial state of lepton polarization and sum over the quark colors. For the sake of simplicity we now set $c_{\mu\nu}^Q = c_{\mu\nu}^U = c_{\mu\nu}^D \equiv c_{\mu\nu}^q$, then the partial amplitudes take the following forms

$$\sum_{s.c.} 2\mathcal{M}_Z^\dagger \delta\mathcal{M}_Z = \frac{2N_c e^4}{\sin^4 2\theta_W} \frac{c_{\mu\nu}^q}{(s - M_Z^2)^2} \left((C_L^{q2} + C_R^{q2})(C_L^{l2} + C_R^{l2})L_V^{\mu\nu} + (C_L^{q2} - C_R^{q2})(C_L^{l2} - C_R^{l2})L_A^{\mu\nu} \right), \quad (6)$$

$$\sum_{s.c.} 4\mathcal{M}_\gamma^\dagger \delta\mathcal{M}_Z = \frac{2N_c e^4 Q_q Q_l}{\sin^2 2\theta_W} \frac{c_{\mu\nu}^q}{s(s - M_Z^2)} \left((C_L^q + C_R^q)(C_L^l + C_R^l)L_V^{\mu\nu} + (C_L^q - C_R^q)(C_L^l - C_R^l)L_A^{\mu\nu} \right), \quad (7)$$

$$\sum_{s.c.} 2\mathcal{M}_\gamma^\dagger \delta\mathcal{M}_\gamma = 2N_c 2e^4 Q_q^2 Q_l^2 \frac{1}{s^2} 2c_{\mu\nu}^q L_V^{\mu\nu}, \quad (8)$$

where $L_{\mu\nu}^V$ and $L_{\mu\nu}^A$ are the vector and the axial Lorentz violating tensors respectively, which depend on the 4-momenta of the incoming and produced particles $e^-(p_1)e^+(p_2) \rightarrow q(k_1)\bar{q}(k_2)$:

$$L_{\mu\nu}^A = ((p_2 k_1)^2 - (p_2 k_2)^2)g_{\mu\nu} - (p_2 k_1)(k_{2\mu} p_{1\nu} + k_{1\mu} p_{2\nu}) + (p_2 k_2)(k_{1\mu} p_{1\nu} + k_{2\mu} p_{2\nu}), \quad (9)$$

$$L_{\mu\nu}^V = (p_2 k_1)(k_{1\nu} p_{2\mu} + k_{2\nu} p_{1\mu}) + (p_2 k_2)(k_{1\nu} p_{1\mu} + k_{2\nu} p_{2\mu}) - (p_1 p_2)(k_{2\mu} k_{1\nu} + k_{1\mu} k_{2\nu} + p_{2\mu} p_{1\nu} + p_{1\mu} p_{2\nu}) + g_{\mu\nu}(p_1 p_2)^2. \quad (10)$$

In Sec. 5 and Sec. 6 we compare the matrix element (5) to the SM expectation in order to estimate the contribution of LV coefficients $c_{\mu\nu}^q$ to the production rate of $q\bar{q}$ pair at the lepton-lepton colliders.

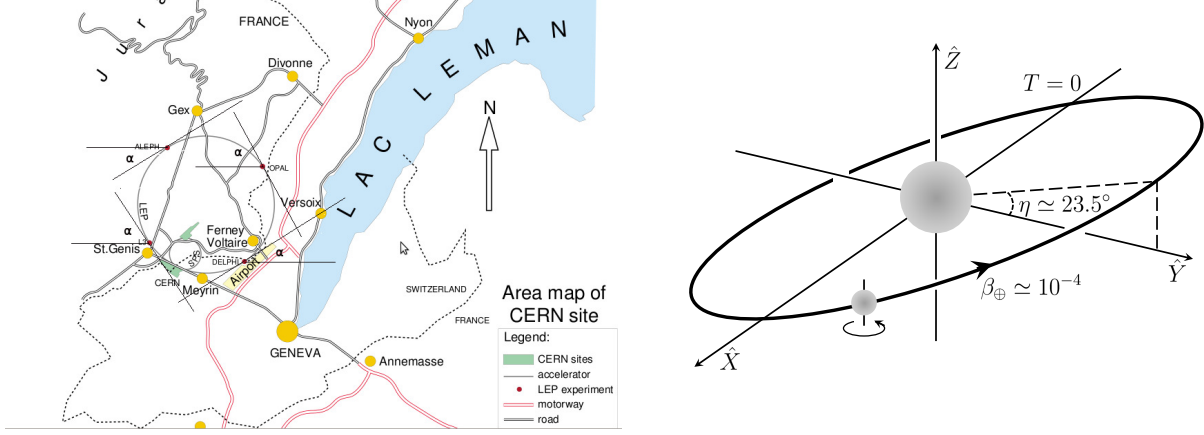


Figure 2: Left panel: orientation of the beam direction for ALEPH and OPAL detectors. Right panel: schematic illustration of the Sun-centered and Earth-based reference frames.

	ALEPH	OPAL	L3	DELPHI
Beam orientation (α)	33.92°	54.50°	55.60°	34.87°
Colatitude (χ)	43.77°	43.77°	43.77°	43.77°

Table 1: The location of the LEP detectors at Earth-based reference frame.

4 The reference frame transformation

If one takes into account the Earth's rotation effect, then we should replace $c_{ij}^q \rightarrow c_{ij}^q(t) = c_{IJ}^q R_i^I(t) R_j^J(t)$ in Eqs. (6-8), the indices I and J numerate the coordinates of the Sun-centered frame, $I, J = (X, Y, Z)$; the indices i and j are associated with Earth-based reference frame (see e.g. Fig. 2 for details). For the sake of simplicity we set also $c_{TT}^q = c_{TI}^q = c_{IT}^q = 0$ throughout the paper. We assume that the relative velocity of Sun-centered and Earth-based reference frames is negligible, so the transformation operation involves only rotations. The explicit form of the rotation matrix $\hat{R}(t) = R_i^J(t)$ is given by the following partial transformations

$$\hat{R}(t) = R_z(\omega t) R_y(\chi) R_x(\pi/2) R_y(\alpha). \quad (11)$$

The corresponding matrices, $R_x(\phi)$, $R_y(\theta)$ and $R_z(\psi)$ are defined by the following way

$$R_x(\phi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{pmatrix}, R_y(\theta) = \begin{pmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{pmatrix}, R_z(\psi) = \begin{pmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

here $\omega = 2\pi/T_{sid}$ is related to the sidereal period, $T_{sid} = (23 \text{ h}, 56 \text{ m}, 4.091 \text{ s})$, χ is the colatitude of the detector, $\chi = (90^\circ - \text{Latitude})$, and α is the angle between the lepton beam and detector's longitude. One can see from Eqs. (6-8) that SME amplitudes squared have the terms which are proportional to the vector part, $c_{IJ}^q R_i^I(t) R_j^J(t) L_{ij}^V$, and to the axial part, $c_{IJ}^q R_i^I(t) R_j^J(t) L_{ij}^A$. Thus the effect of Earth's rotation will introduce a time dependence in the SME contribution, $\delta|\mathcal{M}|_{SME}^2(t)$, to the production rate of $q\bar{q}$ pair.

5 $q\bar{q}$ pair production at LEP

The differential cross-section for $q\bar{q}$ pair production at LEP including Lorentz-violating contribution from SME can be written in the following form

$$\frac{d\sigma}{d\Omega}(e^+e^- \rightarrow q\bar{q}) = \frac{1}{64\pi^2 s} \sum_{s.c.} \left(|\mathcal{M}|_{SM}^2 + \delta |\mathcal{M}|_{SME}^2(t) \right). \quad (12)$$

In this section we restrict our analysis to the case

$$c_{IJ}^q = \begin{pmatrix} c_{XX}^q & c_{XY}^q & c_{XZ}^q \\ c_{YX}^q & c_{YY}^q & c_{YZ}^q \\ c_{ZX}^q & c_{ZY}^q & c_{ZZ}^q \end{pmatrix}. \quad (13)$$

The traceless condition for c_{IJ} requires that $c_{XX}^q + c_{YY}^q = -c_{ZZ}^q$. In order to estimate the collider sensitivity to SME coefficient we average $\delta |\mathcal{M}|_{SME}^2(t)$ over the sidereal period, T_{sid} . The explicit calculation revealed that time-averaged SME amplitude is proportional to SM matrix element squared

$$\langle \delta |\mathcal{M}|_{SME}^2(t) \rangle_t = \frac{1}{T_{sid}} \int_0^{T_{sid}} \delta |\mathcal{M}|_{SME}^2(t) dt = C_{SME} \cdot |\mathcal{M}|_{SM}^2 \quad (14)$$

where

$$C_{SME} = \frac{c_{ZZ}^q}{8} (1 + 3(\cos 2\alpha + \cos 2\chi - \cos 2\alpha \cos 2\chi)). \quad (15)$$

Which means that SME coefficients contribute to the signal cross-section up to the multiplicative factor in the following way

$$\sigma_{e^+e^- \rightarrow q\bar{q}}^{SME} = \sigma_{e^+e^- \rightarrow q\bar{q}}^{SM} \cdot (1 + C_{SME}). \quad (16)$$

It must be point out that after time-averaging (14) our analysis is not sensitive to XY , XZ or ZY elements of (13). So we can constrain only ZZ component of SME coupling. Since ALEPH and OPAL detectors at LEP measured directly the production rate of $q\bar{q}$ events, from experimental uncertainties on $\sigma_{e^+e^- \rightarrow q\bar{q}}^{SM}$ we can derive the limits on $|c_{ZZ}^q|$ under assumption $|c_{ZZ}^u| = |c_{ZZ}^d| = |c_{ZZ}^s| = |c_{ZZ}^c| = |c_{ZZ}^b| = |c_{ZZ}|$, Tab. 2 shows relevant constraints. Beyond this assumption, namely for $|c_{ZZ}^u| \neq |c_{ZZ}^d| \neq |c_{ZZ}^s| \neq |c_{ZZ}^c| \neq |c_{ZZ}^b|$, the systematic uncertainty of $b\bar{b}$ - and $c\bar{c}$ - pairs fraction in total $q\bar{q}$ production needs to be taken into account. It follows from $\sigma_{b\bar{b}(c\bar{c})} = R_{b(c)} \cdot \sigma_{q\bar{q}}$ that the relative uncertainties on $b\bar{b}(c\bar{c})$ cross-section can be expressed in the following way $\Delta\sigma_{b\bar{b}(c\bar{c})}/\langle\sigma_{b\bar{b}(c\bar{c})}\rangle = \Delta\sigma_{q\bar{q}}/\langle\sigma_{q\bar{q}}\rangle + \Delta R_{b(c)}/\langle R_{b(c)}\rangle$. In this case conservative bounds can be found in Tab. 3 for ALEPH and OPAL detectors.

6 The prospects of SME probes in quark sector

In this section we briefly discuss a possible implications of the SME phenomenology for collider experiments and for low energy searches of LV. In contrast to the Sec. 5 now we consider the

	ALEPH	OPAL
$\Delta\sigma_{q\bar{q}}/\langle\sigma_{q\bar{q}}\rangle$	0.78%, see Tab. 4 of Ref. [16]	1.21%, see Tab. 5 of Ref. [18]
$ c_{ZZ} $	< 0.027	< 0.036

Table 2: Conservative bounds on LV coupling of all quarks assuming $|c_{ZZ}^u| = |c_{ZZ}^d| = |c_{ZZ}^s| = |c_{ZZ}^c| = |c_{ZZ}^b| = |c_{ZZ}|$.

	ALEPH	OPAL
$\Delta\sigma_{q\bar{q}}/\langle\sigma_{q\bar{q}}\rangle$	0.78%, see Tab. 4 of Ref. [16]	2.2%, see Tab. 2 of Ref. [19]
$\Delta R_b/\langle R_b\rangle$	9.2%, see Sec. 7.1 of Ref. [16]	13.5%, see Sec. 2.2 of Ref. [19]
$\Delta R_c/\langle R_c\rangle$	10.8%, see Sec. 7.2 of Ref. [16]	-
$ c_{ZZ}^b $	< 0.35	< 0.46
$ c_{ZZ}^c $	< 0.4	-

Table 3: Conservative bounds on LV coupling of c - and b -quarks.

effects of time-dependence in the LV cross-section (12). In order to minimize the number of parameters to be constrained we choose the following benchmark matrix of LV dimensionless couplings

$$c_{IJ}^q = \begin{pmatrix} c_{XX}^q & 0 & 0 \\ 0 & -c_{XX}^q & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

In this case the cross-section for $q\bar{q}$ production can be presented as the cross-section of SM process, $\sigma_{e^+e^- \rightarrow q\bar{q}}^{SM}$, modulated by a time dependent function

$$\sigma_{e^+e^- \rightarrow q\bar{q}}^{SME} = \sigma_{e^+e^- \rightarrow q\bar{q}}^{SM} \cdot (1 + \epsilon(t)), \quad (17)$$

where the contribution of LV couplings is given by the following function

$$\epsilon(t) = c_{XX}^q \left(\cos 2\omega t \{ \cos^2 \alpha \cos^2 \chi - \sin^2 \alpha + \cos^2 \chi \} - \cos \chi \sin^2 \alpha \sin 2\omega t \right). \quad (18)$$

The expression (17) describes the variation of $q\bar{q}$ production signal twice with the sidereal day due to the Lorentz violating contribution of the SME. We leave Monte-Carlo simulation of the signal (17) for the future study.

The analyses of Refs. [12, 13] are very sophisticated and comprehensive test of SME. However, in the light of prospect study, it is instructive to probe SME for the low energy observables [20] as well as for the phenomenological quantities at the high-energy scales [21]. Indeed, Lorentz-violation in quark sector (3) affects the photon polarization operator. Moreover, one can show that Lorentz-violating kinetic term of quark Lagrangian modifies the dispersion relation of quarks at the tree level as well as the photon's dispersion relation at the one-loop level [20]. This effectively means that the velocity of the photon acquires the additional contribution from the terms which "run" via renormalization-group. Therefore, one can constrain the LV coupling of quarks with a high accuracy from laser experiments by measuring the speed of light. This sophisticated analysis is a subject of our study in the nearest future.

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